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MODELING OF EROSION AND TRANSPORT PROCESSES

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Abstract

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Erosion phenomena are the result of complicated natural processes. The determination of their course, their mathematical expression and predicting erosion phenomena of certain intensities is an important hydrological problem. To obtain a solution, it is necessary to evaluate in detail the erosion factors, which act by the origin and course of erosion processes, to analyse their influences, and to apply on these basis correct conclusions to their complex activity.
Dimensional analysis is a conceptual tool often applied in engineering to understand physical situations involving a mix of different kinds of physical quantities. It is routinely used by physical scientists and engineers to check the plausibility of derived equations and computations. It is also used to form reasonable hypotheses about complex physical situations that can be tested by experiment or by more developed theories of the phenomena.
In this paper the dimensional analysis is used for erosion and transport processes modeling.

Key words: soil loss, water, sedimentation, dimensional analysis

Introduction

Water erosion is a widespread phenomena in Eastern Europe. Surface runoff destroys and removes the topsoil layer and deposits it at the foot of slopes or transports it to streams. Significant losses in the national economy occur; they consist especially in decrease of the valuable topsoil and in the loss of the plants` nutrients by which agriculture is weakened in its production bases. Soil particles washed into streams are the main source of sediments which threaten their profiles, reservoirs, ponds and weirs by silting; thus arises the question of the existence and rentability of hydraulic structures and simultaneously other important questions such as unnecessary inundations, increasing the underground water level in the closed area etc.

Soil loss from intensively agriculturally cultivated lands causes water pollution which is unfavourable especially for water supply reservoirs which become important elements in conveying potable water for the inhabitants in the country. Stream erosion causes unfavourable sediment regime with all its consequences. The most important factors causing the origin and influencing the course of water erosion in watersheds are climatic and hydrologic factors, morphological factors – slope gradient, slope length, shape of the slope, exposure of the slope, soil factors, geological factors, vegetation factors, cropping management and socio-economical factors (Bendíková, Švecová, 2005).

These factors may be used for erosion and transport processes modeling. In engineering the application in designs make much of the use of empirical results from a lot of experiments. This data is often difficult to present in a readable form. Dimensional analysis provides a strategy for choosing relevant data and how it could be presented. This is a useful technique in all experimentally based areas of engineering. If it is possible to identify the factors involved in a physical situation, dimensional analysis can form a relationship between them.

Material and methods

It is necessary to know the loss of soil in the catchments in order to state the rate of water erosion. For determination of soil loss is used number of methods. The well- known is the Universal Soil Loss Equation or its revised version.

The Universal Soil Loss Equation (USLE) predicts the long-term average annual rate of erosion on a field slope based on rainfall pattern, soil type, topography, crop system and management practices. USLE only predicts the amount of soil loss that results from sheet or rill erosion on a single slope and does not account for additional soil losses that might occur from gully, wind or tillage erosion. This erosion model was created for use in selected cropping and management systems, but is also applicable to non-agricultural conditions such as construction sites. The USLE can be used to compare soil losses from a particular field with a specific crop and management system to "tolerable soil loss" rates. Alternative management and crop systems may also be evaluated to determine the adequacy of conservation measures in farm planning.

Five major factors are used to calculate the soil loss for a given runoff profile. Each factor is the numerical estimate of a specific condition that affects the severity of soil erosion at a particular location. The erosion values reflected by these factors can vary considerably due to varying weather, soil, morphological and tillage conditions. Therefore, the values obtained from the USLE more accurately represent long-term averages. Universal Soil Loss Equation (USLE) has form (Wischmeier, Smith, 1978)

$$O = R \times K \times LS \times C \times P \,, \tag{1}$$

where

O represents the potential long-term average annual soil loss in tons per acre (hectare) per year. This is the amount, which is compared to the "tolerable soil loss" limits.

R is the rainfall and runoff factor by geographic location. The greater the intensity and duration of the rain storm, the higher the erosion potential.

K is the soil erodibility factor. It is the average soil loss in tons/acre per unit area for a particular soil in cultivated, continuous fallow with an arbitrarily selected slope length of 72.6 feet and slope steepness of 9%.

LS is the slope length-gradient factor. The LS factor represents a ratio of soil loss under given conditions to that at a site with the "standard" slope steepness of 9% and slope length of 72.6 feet. The steeper and longer the slope, the higher is the risk for erosion.

C is the crop/vegetation and management factor. The C factor can be determined by selecting the crop type and tillage method that corresponds to the field and then multiplying these factors together.

P is the support practice factor. It reflects the effects of practices that will reduce the amount and rate of the water runoff and thus reduce the amount of erosion. The most commonly used supporting cropland practices are cross slope cultivation and contour farming.

With additional research, experiments, data, and resources become available, research scientists continue to improve USLE, which led to the development of Revised Universal Soil Loss Equation (RUSLE). RUSLE has the same formula as USLE, but has several improvements in determining factors. These include some new and revised isoerodent maps; a time-varying approach for soil erodibility factor; a subfactor approach for evaluating the cover-management factor; a new equation to reflect slope length and steepness; and new conservation-practice values (IWR, 2002).

Results

For determination of soil loss in watershed by using dimensional analysis is essential to state parameters which characterize water erosion in river basin and is possible to measure them. From the known state of art the following variables are selected (Henriczy, 2006):

- rainfall intensity i [m.s^{-1}]
- soil density ρ [kg.m^{-3}]
- area of the plot S [m^2]
- length of the plot L [m]
- filtration coefficient K [m.s^{-1}]
- vegetation factor (from USLE) C [-]
- coefficient of the slope steepness S_f [-]
- soil loss (from plot) O [kg/m^2.s].

All the given variables are presented in basic dimensions, what is the condition for dimensional analysis application. All these variables are measurable or it is possible to state them. It is rather complex issue to measure soil loss exactly. Results of soil losses obtained from RUSLE (Jakubíková, 2004) are used for development of the model based on dimensional analysis.

Developed model describing water erosion in watershed is based on formation of dimensionless arguments π_i from the stated variables influencing erosion. Their valuable property is that in all existing systems of units they have the same numerical size and they have no dimension. Formation of the model consists in derivation of functional dependence from the expressed dimensionless variables, which in general has always exponential character. Transformation of this function into logarithmic coordinates corresponds to linear character that makes the work with model easier and enables to determine the parameters of linear function (Čarnogurská, 1998).

The general relation among the selected variables, which can effect the erosion in watershed, can be expressed in form

$$\varphi\,(i, \rho, S, L, K, C, S_f, O) = 0. \tag{2}$$

The following equation is valid

$$\pi_i = i^{x_1} \cdot L^{x_2} \cdot S^{x_3} \cdot \rho^{x_4} \cdot O^{x_5} \cdot K^{x_6} \cdot C^{x_7} \cdot S_f^{x_8}\,. \tag{3}$$

The dimension of velocity occurs twice among selected variables therefore the rate K/i – is the rate of two variables with the same dimension and presents dimensionless argument – simplex that will be included to the solution. Dimensionless variables S_f and C present simplexes, too.

The dimensional matrix-relation (4) for basic units has the rank of matrix $m = 3$ and its lines are dimensionally independent on themselves. The form of the matrix is

	i	L	S	ρ	O
kg	0	0	0	1	1
m	1	1	2	–3	–2
s	–1	0	0	0	–1

(4)

From n = 5 independent variables at rank of the matrix m, can be set up i = n – m that is 2 dimensionless arguments π.

Because the number of the unknown parameters $x_i > m$, it is impossible to determine them clearly. The matrix (4) is divided to two parts as well as unknown variables x_i. Then it is modified for solution in the way that its determinant is not equal to zero. The form of the matrix (4) is following

$$A \,.\, B = (-1) \,.\, D \,.\, E \tag{5}$$

The equation (5) has the form (6) after modification

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & -3 \\ -1 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x_1 \\ x_2 \\ x_4 \end{vmatrix} = (-1) \cdot \begin{vmatrix} +1 & 0 \\ -2 & 2 \\ -1 & 0 \end{vmatrix} \cdot \begin{vmatrix} x_5 \\ x_3 \end{vmatrix} \tag{6}$$

The determinant is calculated according to Laplace's equation

$$\Delta_A = \Sigma\, (-1)^{i+j} .\, a_{ij} \,.\, M_{ij} \tag{7}$$

where a_{ij} – the element in rank i and column j,

M_{ij} – subdeterminant of the matrix.

The determinant of the matrix is $\Delta_A = 1$.

The choice of the unknown parameters x_5 a x_3 is done twice, while the selections are independent. The matrix of the selections has the form

	x_5	x_3
1. choice	1	0
2. choice	0	1

(8)

Its determinant $\Delta = 1$, so the condition of the solvability is completed.

It is reached the following according (4) by matrixes multiplying

$$[A]_{3x3} \,.\, [B]_{3x1} = [C]_{3x1} \tag{9}$$

$$[D]_{3x2} \,.\, [E]_{2x1} = [F]_{3x1} \tag{10}$$

According to (5) is valid

$$[C]_{3x1} = (-1) \,.\, [F]_{3x1} \tag{11}$$

The type of the matrixes in relation (11) is the same, so for the simple elements according the matrix (6) is valid the system of three linear equations with five unknown parameters

$$\begin{gathered} x_4 = -x_5 \\ x_1 + x_2 - 3\,x_4 = 2\,x_5 - 2\,x_3 \\ -x_1 = x_5 \end{gathered} \tag{12}$$

Two independent vectors (13) are obtained by solution of the system of linear equations (12)

	x_1	x_2	x_3	x_4	x_5
π_1	–1	0	0	–1	1
π_2	0	–2	1	0	0

(13)

Two complex dimensionless arguments are corresponded to this solution in forms

$$\pi_1 = O^1 .\, i^{-1} .\, \rho^{-1} \tag{14}$$

$$\pi_2 = S^1 .\, L^{-2} \tag{15}$$

The third argument is the rate of two variables with the same dimension

x_6	x_1
1	–1

(16)

The form of dimensionless argument is

$$\pi_3 = K^1 .\, i^{-1} \tag{17}$$

The next dimensionless arguments are

$$\pi_4 = S_f \tag{18}$$

$$\pi_5 = C \tag{19}$$

The searched dimensional homogeneous function described loss of soil in dimensionless form

$$\varphi\,(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0 \tag{20}$$

After adjustment and backward transformation of dimensions for the particular variables, the function has the form

$$ö\left(\frac{O}{i \cdot \rho}, \frac{S}{L^2}, \frac{K}{i}, S_f, C\right) = 0 \tag{21}$$

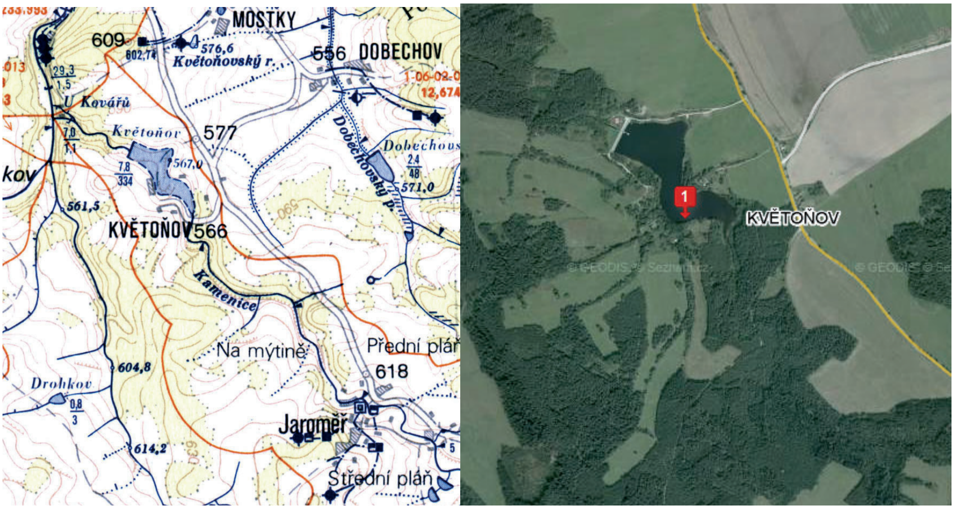

Fig. 1. Situation of the basin Květoňov.

Dimensionless argument π_1 contain the unknown parameter O therefore this argument can be expressed like the function of the other arguments in the form

$$\pi_1 = \varphi\,(\pi_2, \pi_3, \pi_4, \pi_5) \tag{22}$$

Now we are searching the dependence of arguments π_1 and π_2. The relation between independent argument π_2 and dependent argument π_1 can be define by exponential equation

$$\pi_1 = A \cdot \pi_2^{B} \tag{23}$$

The real course of the dependence (23) of dimensionless arguments π_1 to π_2 in logarithmic coordinates is depicts in Fig. 1. The relation (23) is transformed to linear dependence by logarithmic calculation

$$\log \pi_1 = \log A + B \,.\, \log \pi_2 \tag{24}$$

Regression straight line for the calculation of the regression coefficients can be calculated by the method of least squares and relevant is

$$a = \frac{\sum x_i^2 \cdot \sum y_i - \sum x_i \cdot \sum x_i \cdot y_i}{n \cdot \sum x_i^2 - \left(\sum x_i\right)^2} \tag{25}$$

$$b = \frac{n \cdot \sum x_i \cdot y_i - \sum x_i \cdot y_i}{n \cdot \sum x_i^2 - \left(\sum x_i\right)^2}, \tag{26}$$

where $\sum x_i = \sum \log \pi_2$

$\sum y_i = \sum \log \pi_1$

because $$a = \log A \quad A = 10a \tag{27}$$

$$B = b \tag{28}$$

After completing of the relation (23) is obtained the equation

$$\frac{O}{i \cdot \rho} = A \cdot \left(\frac{S}{L^2} \right)^{B} \tag{29}$$

After modification of the relation (29) the following equation is valid

$$O = A \cdot S^{B} \cdot i \cdot \rho \cdot L^{-2B} \tag{30}$$

Considering other dimensionless arguments has the relation characterizing soil loss in river basin final form

$$O = A \cdot S^{B} \cdot i \cdot \rho \cdot L^{-2B} \cdot X \cdot Y \cdot Z, \tag{31}$$

where X, Y, Z are dependences of the other dimensionless arguments to variables defined in the next part.

Relation (31) presents the model of soil loss in watershed.

For soil loss assessment the watershed Kamenice was chosen. Kamenice is situated in the southern Bohemia. The main water course in the river basin is Kamenice river that catches water from the watershed into the basin Květoňov. The area of the river basin is 32.15 km^2. It is spread in broken relief. The soil in the watershed is used mainly for agriculture (Jakubíková, 2004). The situation of the basin Květoňov is shown in Fig. 1. The obtained values of the relevant parameters are introduced in Table 1. The plots (1–28) near river banks grassed or agriculturally used are considered.

According to equations (14) and (15) the dimensionless arguments π_1 and π_2 are calculated. The values of these arguments are presented in Table 2. As well as the values of soil losses are presented in Table 2. Soil losses are calculated from developed model based on dimensionless analysis (O_{model}). These values are compared with soil losses calculated from existing Universal Soil Loss Equation (O_{RUSLE}). The differences between the values O_{RUSLE} and O_{model} are expressed as uncertainty, which is computed from equation

$$\sigma = \frac{1}{n} \cdot \sum_{i=1}^{n} \frac{\left| O_{RUSLE} - O_{model} \right|}{O_{RUSLE}} \cdot 100 \tag{32}$$

The average value of the uncertainty for this case is 26.1%.

Fig. 2. depicts the dependency of dimensionless arguments π_1 and π_2. The regression equation is in the form

$$y = 0.0001.\, x^{0.1268}, \tag{33}$$

where y is independent argument π_1
x is dependent argument π_2

The regression coefficients are then $A = 0.0001$ and $B = 0.1268$.

T a b l e 1. Values of relevant arguments.

Plot	S [m^2]	L [m]	i [m/s]	ρ [kg/m^3]	K [m/s]	C [-]	S_f [-]
1	25 000	110	2.63E-08	1880	4.0E-06	0.005	1.7536
2	15 000	100	2.63E-08	1880	4.0E-06	0.005	1.3508
3	255 000	970	2.63E-08	1850	7.0E-06	0.610	1.0016
4	72 000	290	2.63E-08	1850	7.0E-06	0.610	1.4117
5	71 000	260	2.63E-08	1850	7.0E-06	0.005	1.1689
6	132 000	700	2.63E-08	1850	1.5E-05	0.610	0.7357
7	48 000	290	2.63E-08	1850	7.0E-06	0.610	1.0000
8	289 000	1460	2.63E-08	1840	2.5E-06	0.610	1.2113
9	29 000	180	2.63E-08	1850	7.0E-06	0.005	0.5712
10	53 000	560	2.63E-08	1850	7.0E-06	0.092	0.4778
11	65 000	350	2.63E-08	1850	7.0E-06	0.005	3.1411
12	59 000	240	2.63E-08	1850	7.0E-06	0.005	2.0019
13	39 000	160	2.63E-08	1850	7.0E-06	0.005	3.2743
14	50 000	500	2.63E-08	1850	1.5E-05	0.005	0.4544
15	46 000	160	2.63E-08	1850	1.5E-05	0.005	1.7730
16	118 000	950	2.63E-08	1870	1.5E-05	0.005	1.2783
17	54 000	370	2,63E-08	1850	1.0E-05	0.005	1.5781
18	34 000	130	2,63E-08	1900	2.5E-06	0.005	1.7536
19	37 000	120	2.63E-08	1850	7.0E-06	0.092	1.9745
20	245 000	620	2.63E-08	1850	1.5E-05	0.092	1.3735
21	60 000	450	2.63E-08	1900	2.5E-06	0.005	1.1689
22	34 000	130	2.63E-08	1850	1.5E-05	0.050	0.8440
23	126 000	650	2.63E-08	1900	2.5E-06	0.005	1.0294
24	412 000	2400	2.63E-08	1850	7.0E-06	0.092	1.2710
25	34 000	130	2.63E-08	1850	7.0E-06	0.005	1.5457
26	171 000	750	2.63E-08	1850	7.0E-06	0.005	1.8802
27	99 000	460	2.63E-08	1850	7.0E-06	0.005	1.1066
28	231 000	1520	2.63E-08	1800	1.0E-05	0.005	0.8188

The values of soil loss are calculated according to relation (31) on the base of relevant input parameters and calculated coefficients *A* and *B*. The final form of developed model is

$$O = 0,000123 \cdot S^{0.12684} \cdot i \cdot \rho \cdot L^{-0.25368} \cdot X \cdot Y \cdot Z\,, \qquad (34)$$

where $X = 0,0011 \cdot \left(\frac{K}{i}\right)^2 - 0,8284 \cdot \left(\frac{K}{i}\right) + 226,14$

$Y = 0,0455 \cdot C^2 - 0,0311 \cdot C + 0,0072$

$Z = 0,2312 \cdot S_f^2 - 0,1573 \cdot S_f + 0,5882$

T a b l e 2. Values of dimensionless arguments, calculated soil loss, uncertainty.

Plot	π_1	π_2	O_{RUSLE} [kg/m^2.s]	O_{model} [kg/m^2.s]	σ [%]
1	1.35E-04	2.066116	6.681E-09	9.707E-09	45.289
2	1.01E-04	1.500000	4.975E-09	7.234E-09	45.413
3	9.77E-05	0.271017	4.756E-09	3.832E-09	19.430
4	1.12E-04	0.856124	5.466E-09	5.764E-09	5.450
5	9.46E-05	1.050296	4.607E-09	4.030E-09	12.526
6	7.70E-05	0.269388	3.748E-09	4.322E-09	15.316
7	1.07E-04	0.570749	5.190E-09	4.207E-09	18.943
8	1.24E-04	0.135579	5.999E-09	7.512E-09	25.220
9	4.34E-05	0.895062	2.115E-09	2.735E-09	29.326
10	4.44E-05	0.169005	2.162E-09	2.768E-09	28.018
11	2.63E-04	0.530612	1.282E-08	1.142E-08	10.940
12	1.35E-04	1.024306	6.567E-09	6.756E-09	2.885
13	2.63E-04	1.523438	1.279E-08	1.391E-08	8.688
14	5.01E-05	0.200000	2.438E-09	2.826E-09	15.908
15	1.34E-04	1.796875	6.506E-09	8.435E-09	29.639
16	1.26E-04	0.130748	6.202E-09	7.322E-09	18.054
17	1.51E-04	0.394490	7.366E-09	1.084E-08	47.202
18	1.99E-04	2.011834	9.953E-09	1.224E-08	23.033
19	1.46E-04	2.569444	7.109E-09	9.828E-09	38.248
20	2.07E-04	0.637357	1.008E-08	7.553E-08	25.118
21	1.85E-04	0.296296	9.243E-09	6.634E-09	28.224
22	7.44E-05	2.011834	3.624E-09	4.742E-09	30.850
23	1.43E-04	0.298225	7.166E-09	6.073E-09	15.250
24	1.33E-04	0.071528	6.468E-09	4.026E-09	37.753
25	1.24E-04	2.011834	6.043E-09	5.566E-09	7.898
26	1.91E-04	0.304000	9.322E-09	5.385E-09	42.235
27	7.09E-05	0.467864	3.450E-09	3.495E-09	1.304
28	6.29E-05	0.099983	2.980E-09	1.957E-08	34.324

Equation for S_f is obtained from literature (Fulajtár, Janský, 2001; Holý, 1994)

$$S_f = \frac{\left(0{,}43 + 0{,}30 \cdot \alpha + 0{,}043 \cdot \alpha^2\right)}{6{,}613}, \tag{35}$$

where α is slope [%]

The coefficients X, Y, Z present dependences π_3, π_4, π_5 or (K/i, C-factor, S_f) to π_1 according relation (22). These dependences are determined from regression lines by using the software MS Excel.

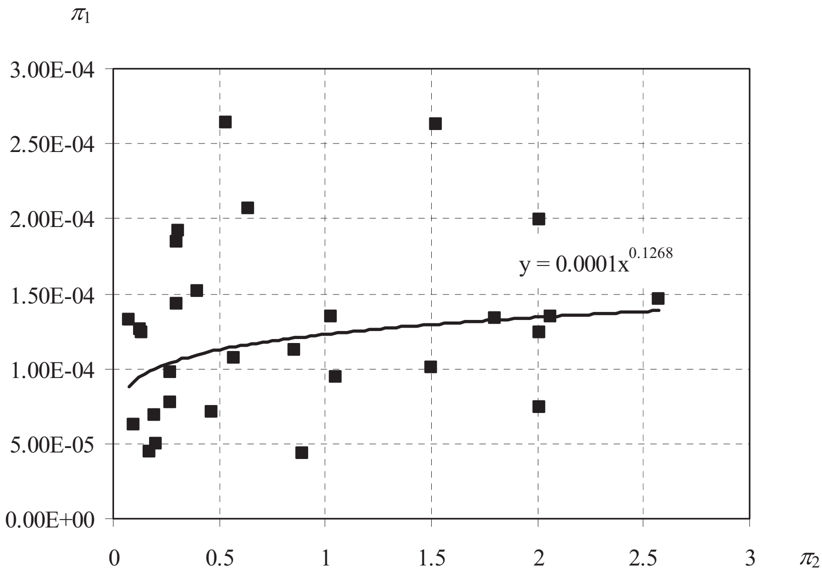

Fig. 2. The real course of the function $\pi_1 = \varphi(\pi_2)$.

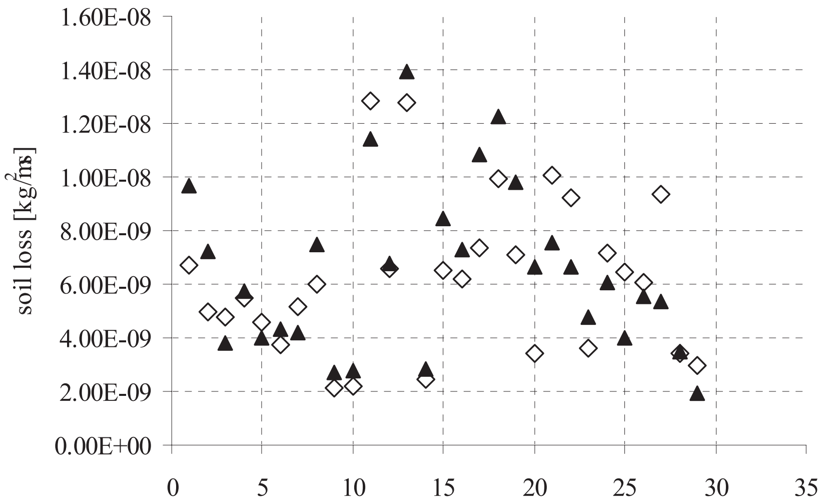

Fig. 3. Comparison of the soil loss values calculated from USLE and from model.

Discussion

Comparison of the soil loss values calculated from USLE and from developed model is shown in the Fig. 3. Differences between calculated values were occurred because of not exactly the same influences considering.

The rate of erosion threat in watershed depends on soil loss (O). Heinige, (1995) states the following:

- no or low erosion is if O [t/ha/year] < 4;
- medium erosion is if O [t/ha/year] 4–8;
- high erosion is if O [t/ha/year] 8–12;
- very high erosion is if O [t/ha/year] >12.

Average soil loss (O_{model}) in this watershed is 9.53 t/ha/year. There is high erosion according to mentioned dividing.

The developed model (34) represents a new approach to soil loss calculation and following this to water erosion determination in watersheds.

Conclusion

Water erosion causes serious problems in watersheds. Movement of sediment and associated agricultural pollutants into watercourses is the major impact resulting from erosion. This leads to sedimentation in watercourses and dams, disruption of the ecosystems of lakes, and contamination of drinking water. A more minor effect can occur in situations where eroded soil has a decreased capacity to absorb water: increased runoff may lead to downstream flooding and local damage to property.

In this paper the dimensional analysis is used for soil loss modeling, as the main impact of water erosion in catchments. Dimensional analysis is a method widely used in environmental engineering as well as in water management. However, there are no references about using dimensionless analysis for modeling of erosion and transport processes in watersheds. The average uncertainty (variation) of the developed model is 26.1%. This is an appropriate value for modeling such phenomena as soil loss in watershed. As was already mentioned, soil loss in watersheds is influenced by numerous factors. The differences between the calculated values O_{RUSLE} and O_{model} could occur mainly because of the variables selection – may be not all the relevant variables were included. It is still possible to use developed model for soil loss determination as well as rate of water erosion statement. It will be valuable to use this model for verification in conditions of Slovakia. Even there is a great requirement for input data.

Translated by the authors

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References

Bendíková, M., Švecová, A., 2005: The soil loss prediction in basin by empirical models using (in Slovak). In 6. konferencia Vplyv vodohospodárskych stavieb na tvorbu a ochranu životného prostredia. Podbanské. ASCO, Bratislava, p. 29–36.

Čarnogurská, M., 1998: Dimensional analysis and similarity theory and modelling in practise (in Slovak). Elfa, Košice, 64 pp.

Fulajtár, E., Janský, L., 2001: Water erosion of soil and erosion protection (in Slovak). VÚPOP, Bratislava, p. 310.

Heinige, V., 1995: Watershed protection and management (in Slovak). STU, Bratislava.

Henriczy, G., 2006: Erosion and transport processes evaluation by using geographical information systems (in Slovak). Diploma thesis. SvF TU, Košice.

Holý, M., 1994: Erosion and environment (in Czech). ČVTU, Praha, 383 pp.

Institute of Water Research (IWR), 2002: About RUSLE. http://www.iwr.msu.edu/rusle/about.htm

Jakubíková, A., 2004: Using of RUSLE software for erosion endangerment determination in condition of Czech Republic (in Czech). Thesis. Fakulta stavební ČVUT, Praha.

Wischmeier W.H., Smith, D.D., 1978: Predicting rainfall erosion losses. A guide to conservation planning. Agriculture handbook. U.S Department of Agriculture.